



NES/MAA  
Collegiate Mathematics  
Competition 2006

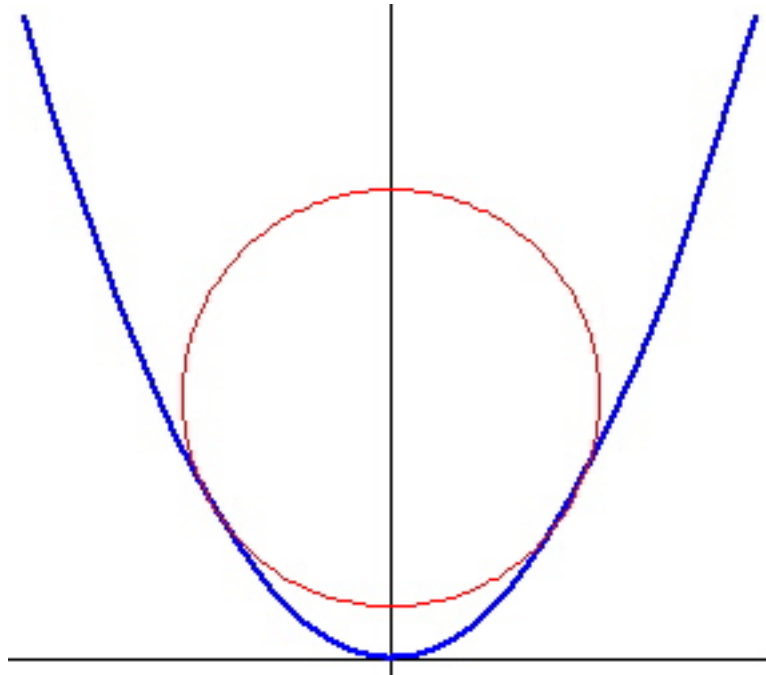
Team Number: \_\_\_\_\_

Sacred Heart University

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# NES/MAA Collegiate Mathematics Competition 2006

1. The figure below shows a circle with radius 1 inscribed in the parabola  $y = x^2$ . Find the center of the circle.



2. Which number is bigger:  $e^\pi$  or  $\pi^e$ ? (Your calculator will of course tell you the answer; you need to prove it.)
3. If  $x$ ,  $y$ , and  $z$  are positive numbers, prove that

$$\frac{(x^2 + 1)(y^2 + 1)(z^2 + 1)}{xyz} \geq 8.$$

4. The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hands changing at one o'clock?
5. Let  $A$  be a square matrix and suppose that there exist positive integers  $m$  and  $n$  such that  $A^m = I$  and  $A^n \neq I$ . Calculate

$$\det(I + A + A^2 + \cdots + A^{m-1}).$$

6. Can a group be a union of two proper subgroups?
7. A chicken and a half can lay an egg and a half in a day and a half. How long will it take for two chickens to lay 32 eggs?