

A decent try at all of the problems can earn you 5 points on Exam #2. (Less-than-decent tries earn less than 5 points – my call.) These are due at the exam on Wednesday.

1. The point $(4, 2)$ lies on the curve $y = \sqrt{x}$.
- (a) If Q is the point (x, \sqrt{x}) , use a calculator to find the slope of the secant line PQ (correct to six decimal lines) for the following values of x :

$$x = 5, 4.5, 4.1, 4.01, 4.001$$

$$x = 3, 3.5, 3.9, 3.99, 3.999$$

- (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(4, 2)$.
- (c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(4, 2)$.
- (d) Use one of the two formulas for the slope of the tangent line and verify your guess in part (b) is correct.
2. If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after t seconds is given by $h = 58t - 0.83t^2$.
- (a) Find the average velocity over the following time intervals:

$$[1, 2], [1, 1.5], [1, 1.1], [1, 1.01], [1, 1.001]$$

- (b) Find the instantaneous velocity after 1 s.
3. State the definition of the derivative.
4. Determine $f'(x)$ directly from the definition of the derivative:

(a) $f(x) = x^3 + 5x + 4$

(b) $f(x) = \sqrt{3 - 5x}$

(c) $f(x) = \frac{4 - x}{3 + x}$

5. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.
- (a) Find the velocity and acceleration functions.
- (b) When is the particle moving upward and when is it moving downward?
- (c) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.

6. Calculate y' .

(a) $y = (x + 2)^8(x + 3)^6$

(b) $y = \frac{x}{\sqrt{9 - 4x}}$

(c) $x^2y^3 + 3y^2 = x - 4y$

(d) $y\sqrt{x-1} + x\sqrt{y-1} = xy$

(e) $y = \sin(\cos x)$

(f) $y = \sqrt{\sin \sqrt{x}}$

(g) $y = \cot(3x^2 + 5)$

(h) $x \tan y = y - 1$

7. Find the equation of the tangent to the curve $y = \frac{x}{x^2 - 2}$ at $(2, 1)$.

8. The volume of a cube is increasing at a rate of $10 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 10 cm ?
9. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of $2 \text{ cm}^3/\text{s}$, how fast is the water level rising when the water is 5 cm deep?
10. A balloon is rising at a constant speed of 5 ft/s . A boy is cycling along a straight road at a speed of 15 ft/s . When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?