



NES/MAA
Collegiate Mathematics
Competition 2008

Team Number: _____

Bentley University

November 21, 2008

NES/MAA Collegiate Mathematics Competition 2008

1. Find the exact value of

$$\int_0^1 \frac{4x^3(1+x^{4(2007)})}{(1+x^4)^{2009}} dx.$$

2. A boy goes off to college and after the first semester he's run out of money. In fact, he's so broke he doesn't have enough money to call home, or to even send a letter. However, he manages to find a postcard with a stamp already on it.

He writes the following message:

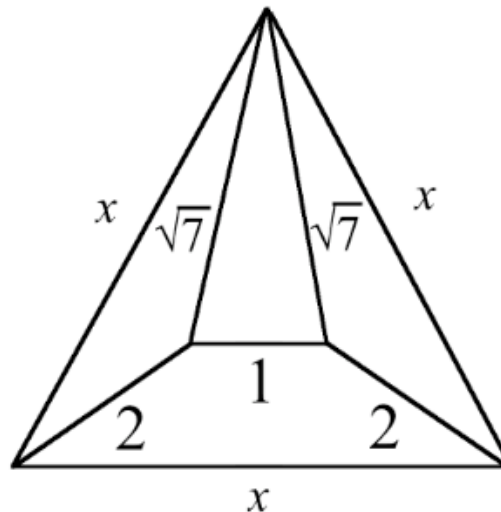
$$\begin{array}{rcccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline & & M & O & N & E & Y \end{array}$$

Assuming his parents are able to decipher the code (and they don't tell him to drop dead), how much money did they send the boy?

3. Solve the equation for x in terms of c :

$$2 \log_x c - \log_{cx} c - 3 \log_{c^2x} c = 0.$$

4. The increasing sequence $S = \{2, 3, 5, 6, 7, 10, 11, \dots\}$ consists of all positive integers which are neither a perfect square nor a perfect cube. What is the 500th term of S ?
5. Alice, Bob, and Carol repeatedly take turns tossing a fair regular six-sided die. Alice begins; Bob always follows Alice; Carol always follows Bob; and Alice always follows Carol. Find the probability that Carol will be the first to toss a six.
6. Determine x in the following equilateral triangle:



7. Snow begins to fall during the morning of November 21 and continues steadily into the afternoon. At noon a snowplow begins removing snow from a road at a constant rate. The plow travels 6 km from noon to 1 p.m. but only 3 km from 1 p.m. to 2 p.m. To the nearest minute, when did the snow begin to fall? [Hint: Let t be the time measured in hours after noon; let $x(t)$ be the distance traveled by the plow at time t . Then the speed of the plow is dx/dt . Let b be the number of hours before noon that it began to snow. Find an expression for the height of the snow at time t . Then use the given information that the rate of removal R (in m^3/h) is constant.]