



NES/MAA
Collegiate Mathematics
Competition 2007

Team Number: _____

Framingham State College

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1. Calculate exactly (*i.e.*, no decimals) $\lim_{x \rightarrow 3} \frac{x}{x-3} \left(\int_3^x \sin t \, dt \right)$.
2. Determine the value of

$$S = \log(\tan 1^\circ) + \log(\tan 2^\circ) + \cdots + \log(\tan 88^\circ) + \log(\tan 89^\circ).$$

3. Experiments have determined that when a particular steel ball is bounced on a hard surface, it bounces to half its original height. For example, if it is dropped from a height of 6 feet, it will bounce to 3 feet. Assuming that the ball obeys this law exactly, for what length of time will the ball continue to bounce if it is dropped from a height of 16 feet? (Or will it bounce forever?)

[Recall from calculus that since the acceleration due to gravity is 32 ft/sec^2 , an object falling to the ground from height h (in feet) or bouncing from the ground to height h requires $\sqrt{h}/4$ seconds to do so.]

4. Prove that a group G of order 15 must be cyclic.
5. An object moves 8 cm in a straight line from A to B , turns at an angle α , measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C . What is the probability that $AC < 7$?
6. A and B are positive integers in the decimal system such that
 - $A = 7B$ and
 - the sum of the digits of A is twice the sum of the digits of B .

If C is the number formed by writing the digits of B immediately after the digits of A , prove that C is a multiple of 9.

7. A group of 11 scientists wants to design a cabinet in which to keep their top secret papers. They propose to outfit the cabinet with a set of locks and supply a certain number of keys to each scientist so that the cabinet can be opened only when a majority of the scientists is present. That is,
 - no group of 5 or fewer is to be able to open all the locks, and
 - every group of 6 or more is to have keys for all the locks.

What is the minimum number of locks that must be built into the cabinet and what is the minimum number of keys each scientist must be given?